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Improving Computational and Memory Requirements of Simultaneous Localization and Map Building Algorithms

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Abstract

This paper addresses the problem of implementing simultaneous localisation and map building (SLAM) in very large outdoor environments. A method is presented to reduce the computational requirement from *~O(N^2)* to *~O(N)*, being *N* the states used to represent all the landmarks and vehicle pose. With this implementation the memory requirement are also reduced to *~O(N)*. This algorithm presents an efficient solution to the full update required by the Compressed Extended Kalman Filter algorithm (CEKF). Experimental results are also presented.

1 Introduction

The problem of localization when a map of the environment is available has been solved before with very efficient algorithms [1] [2]. Similarly, there are well proven techniques for the generation of environment maps using observations obtained from known locations [3]. There are also successful real-time implementations of localization loops based on Bayesian approaches [4].

A more challenging problem is the solution of both position and localisation at the same time. This problem is called "simultaneous localization and map building" (SLAM) [5],[6] and "Concurrent Map and Localisation" (CML) [7].

Optimal SLAM approaches based on Bayesian filtering with non-Gaussian assumptions are extremely expensive making them difficult to apply in real time. Nevertheless these methods present some significant advantages such as the inherent solution of the data association problem. In fact they can address the localization problem starting at a completely unknown position and to solve the "kidnapped robot" problem, that is a robot that is suddenly moved to another position without being told. Some approaches based on these techniques [2], approximate the probability representation using samples of the probability density distribution. Efforts to apply these techniques in real time are started to appear. In [8] and [9] a Sum of Gaussian (SOG) distribution is used to build a feature map of landmarks to represent the environment and its application to the full-Bayesian SLAM problem is presented. It is also argued that the SOG method provides a computationally tractable implementation of the full Bayes SLAM.

There are several optimal and sub-optimal techniques that are attractive to solve SLAM in real time, most of them based on the Extended Kalman Filter (EKF) framework, [7],[10] and [11]. These techniques assume Gaussian or at least uni-modal density probability distributions. The major problems of EKF implementations are:

1. The estimation must always have an uni-modal, more strictly a Gaussian, probability distribution.
2. Revisits to known places and external absolute observations such as GPS measurement can produce 'strong' updates, generating incorrect corrections.
3. A large number of map objects will quadratically increase the memory and computational power requirements.

The first two limitations can be overcome by using Hybrid combinations of EKF and non-Gaussian Bayesian filters [12]. The non-Gaussian strategy is applied in the initialisation phase or at any stage when the probability distribution can not be uni-modally approximated, that is when the data association is not possible. In this case the EKF based system will be in fault since the high uncertainty prevents the data association making impossible the incorporation of the observation using the EKF update. Under this condition the usage of those observations will generate multi-hypotheses in the estimations. In [12], the Bayesian estimator is used until sufficient information is obtained to collapse all the multi-hypotheses in a unique uni-modal distribution that the normal EKF filter is able to use.

The third item addresses the computation and memory requirement of full SLAM. It is well known that this algorithm has computational requirements of *~O(N^2)*, being *N* the states used to represent all the landmarks and vehicle pose. In [5] the Compressed EKF (CEKF) was introduced. This algorithm dramatically reduces the computational requirements of SLAM while generating equivalent results to the full standard implementation. This algorithm represents a remarkable improvement when the vehicle operates in a local area. Still a full SLAM update is required when a transition to a different area is made.

Another aspect that has not been addressed is the memory requirement. A system with 10000 states will require up to 800 Mb to maintain the covariance of the states. Both memory and global update calculation have a cost of order *~O(N^2)* as stated before. When using the CEKF the cost in

the global update evaluation is not critical provided the transitions from local areas are not frequent. There are no memory usage improvements that can be obtained with this approach.
The reason of the expensive memory requirements is due to the need of maintaining cross-covariance terms between all the states. This implies that the complete covariance matrix has to be maintained in fast processing memory (RAM). We argue that the maintenance of the complete covariance matrix is important in cases where the cross-correlation between states is strong or at least not negligible. Any attempt to conservatively de-correlate a subset of states implies an increase in the value of some diagonal sub-matrixes. If subsets of lightly correlated states are present then de-correlation of these states can be done with a small loss in the predicted estimation quality. This paper makes use of RLR (Relative Landmark Representation) to generate close to optimal de-correlation that will significantly reduce the computational and memory requirement in large environments.
This work is organized as follows. Section 2 presents a brief introduction to the CEKF filter. The de-correlation algorithm is presented in section 3 and the integration with the CEKF in SLAM applications is described in section 4. Finally experimental results are presented in section 5 with conclusions given in section 6.

2 Optimal Compressed Extended Kalman Filter (CEKF).

This section presents a brief summary of the CEKF algorithm. A full description is presented in [5]. Assume that for a period of time $\Omega = \{k / k_1 \le k \le k_2\}$, the model and observations of system can be expressed in the following form:

$$X = \begin{bmatrix} X_a \\ X_b \end{bmatrix}, \quad X_a \in R^{N_a}, \quad X_b \in R^{N_b},$$

$$\begin{bmatrix} X_a(k+1) \\ X_b(k+1) \end{bmatrix} = \begin{bmatrix} f_a\left(X_a(k), u(k), k\right) + \nu_a(k) \\ X_b(k) + \nu_b(k) \end{bmatrix} \quad \textbf{(1)}$$

$$y(k) = h\left(X_a(k), k\right) + \nu_h(k)$$

$$\forall (X, u, k) \quad / \quad k \in \Omega$$

Where the model and observation noises $\nu_a(k), \nu_b(k), \nu_h(k)$ are Gaussian and uncorrelated.

An EKF filter running during the period Ω can also be evaluated using the “Compressed EKF” algorithm as shown bellow. At the beginning of the period Ω, $k = k_1$, a set of auxiliary matrices are initialized:

$$\phi_{k_1} = I, \quad \psi_{k_1} = \bar{\bar{0}}, \quad \theta_{k_1} = \bar{0}, \quad Q^*_{bb,k_1} = \bar{\bar{0}}$$
$$\left(\phi_k, \psi_k \in R^{N_a x N_a}, \quad \theta_k \in R^{N_a}\right) \quad (2)$$

At every prediction or update stage, during the time period Ω, a normal EKF algorithm is run using the subsystem P_{aa}, X_a. An additional set of matrices is maintained to store the information gathered to be transfered to the rest of the system in the full update. The size of these matrices will always be smaller than P_{aa}. During the period $k_1 \le k \le k_2$ any prediction step is implemented using the standard EKF prediction for the sub-system P_{aa}, X_a and the update of the auxiliary matrices:

$$\phi_k = J_{aa} \cdot \phi_{k-1}, \quad \psi_k = \psi_{k-1}, \quad \theta_k = \theta_{k-1}$$
$$\left(J_{aa} = \partial f_A / \partial X_A\right) \quad (3)$$
$$Q^*_{bb,k} = Q^*_{bb,(k-1)} + Q_{bb,k}$$

An observation is processed with a standard EKF update equations for the sub-system P_{aa}, X_a and the update of the auxiliary matrices as follows:

$$\begin{matrix} \phi_k = (I - \mu_k) \cdot \phi_{k-1} \\ \psi_k = \psi_{k-1} + \phi^T_{k-1} \cdot \beta_k \cdot \phi_{k-1} \end{matrix} \left\{ \begin{matrix} H_{a,k} = \partial h / \partial X_a \\ \beta_k = H^T_{a,k} \cdot S_k^{-1} \cdot H_{a,k} \\ \mu_k = P_{aa,k} \cdot \beta_k \end{matrix} \right. \quad (4)$$

$$\theta_k = \theta_{k-1} + \phi^T_{k-2} \cdot H^T_{a,k-1} \cdot S^{-1}_{k-1} \cdot z_{k-1}$$

At the end of the interval Ω, $k = k_2$, the update of all the states in the system and covariance matrix is performed. This step is called global or full update:

$$P_{ab,(k_2)} = \phi_{(k_2)} \cdot P_{ab,(k_1)}$$
$$P_{bb,(k_2)} = P_{bb,(k_1)} - P_{ba,(k_1)} \cdot \psi_{(k_2)} \cdot P_{ab,(k_1)} + Q^*_{bb,k_2} \quad (5)$$
$$X_{b,(k_2)} = X_{b,(k_1)} - P_{ba,(k_1)} \cdot \theta_{k_2}$$

It can be seen that the knowledge of $X_b(k), P_{bb}(k), P_{ab}(k), P_{ba}(k)$ is only explicit at times $k = k_1$ and $k = k_2$. No explicit information about this family of states and their related covariance and cross-covariance matrices is required at times $k / k_1 < k < k_2$. All this information is implicitly contained (at any instant k) in the matrixes ϕ_k, ψ_k, θ_k. The matrices ϕ_k, ψ_k have dimensions $N_a * N_a$ and the vector θ_k is of order N_a. All the information (covariance and cross-covariance values) related to the states $X_b(k)$ remains compressed in these three auxiliary matrices.
It is important to note that the CEKF estimation is optimal and generates the same results as full EKF implementation.

3 De-correlation algorithm

The maintenance of the complete covariance matrix is important in cases where the cross-correlation between states is strong or at least not negligible. Any attempt to conservatively de-correlate a subset of states implies an increase in the value of some diagonal sub-matrixes. If subsets of weakly correlated states are present then de-correlation of these states can be done with a small loss in the predicted estimated quality.

Unfortunately when a map represents the landmarks in absolute form with respect to a single global frame, all the states are or tend to be strongly correlated. This correlation is of major relevance when a conservative de-correlation procedure is desired. Most of the state's high correlation coefficients are due to the map representation and not due to map estimation problem itself. An appropriate map representation that avoids this problem is the Relative Landmark Representation (RLR). This representation divides the map into sub-regions where the landmarks are defined with respect to local coordinate frames.

For the 2-D case each local frame is defined based on two local landmarks represented in global coordinate. The high correlation characteristic persists but only between the frame base landmarks and the vehicle states. These landmarks represent a small subset of the total landmark population.

An attractive aspect of the RLR is that the cross-correlation between relative landmarks that belong to different frames (or constellations) tends to be extremely low, especially when these constellations are distant. Then a strategy that cancel the weakly cross-correlation terms and maintain the strong cross-correlation terms can be implemented. This sub-optimal simplification will generate results very close to optimal when the RLR representation and appropriate map management strategies are used.

The algorithm to cancel the weakly cross-correlation terms in a consistent manner is now presented.

Given a symmetric nonnegative definite matrix $P \geq 0, \quad P \in R^{2x2}$ it is possible to obtain a de-correlated (diagonal) matrix $D \geq P$ according to

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} =$$

$$= \begin{bmatrix} p_{11} + \kappa \cdot |p_{12}| & 0 \\ 0 & p_{22} + \dfrac{|p_{12}|}{\kappa} \end{bmatrix} - \begin{bmatrix} \kappa \cdot |p_{12}| & -p_{12} \\ -p_{12} & \dfrac{|p_{12}|}{\kappa} \end{bmatrix} = \qquad (6)$$

$$= D - \tau \leq \begin{bmatrix} p_{11} + \kappa \cdot |p_{12}| & 0 \\ 0 & p_{22} + \dfrac{|p_{12}|}{\kappa} \end{bmatrix} = D$$

$$\forall \quad \kappa > 0$$

This is true since τ is a nonnegative definite matrix:

$$\tau = \begin{bmatrix} \kappa \cdot |p_{12}| & -p_{12} \\ -p_{12} & \dfrac{1}{\kappa} \cdot |p_{12}| \end{bmatrix} \geq 0 \qquad \forall \quad \kappa > 0 \qquad (7)$$

In the general case it is possible to de-correlate the covariance matrices corresponding to two groups of states by using a similar technique:

$$P = \begin{bmatrix} \alpha & C \\ C^T & \beta \end{bmatrix} \qquad \begin{matrix} \alpha \in R^{n\,x\,n} \\ \beta \in R^{m\,x\,m} \\ C \in R^{n\,x\,m} \end{matrix}$$

$$C = \begin{bmatrix} c_{11} & .. & .. & c_{1m} \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ c_{n1} & .. & .. & c_{nm} \end{bmatrix} \qquad (8)$$

A conservatively de-correlated block diagonal matrix bound can be obtained for this matrix:

$$P = \begin{bmatrix} \alpha & C \\ C^T & \beta \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} - \begin{bmatrix} 0 & -C \\ -C^T & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} - \begin{bmatrix} \tilde{\alpha} & -C \\ -C^T & \tilde{\beta} \end{bmatrix} + \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\beta} \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha + \tilde{\alpha} & 0 \\ 0 & \beta + \tilde{\beta} \end{bmatrix} - \begin{bmatrix} \tilde{\alpha} & -C \\ -C^T & \tilde{\beta} \end{bmatrix} \leq$$

$$\leq \begin{bmatrix} \alpha + \tilde{\alpha} & 0 \\ 0 & \beta + \tilde{\beta} \end{bmatrix}$$

$$\tilde{\alpha}, \tilde{\beta} \quad / \quad \tau = \begin{bmatrix} \tilde{\alpha} & -C \\ -C^T & \tilde{\beta} \end{bmatrix} \geq 0$$

The matrix τ will be nonnegative definite if the matrices $\tilde{\alpha}$ and $\tilde{\beta}$ are formed with the following expressions:

$$\tilde{\alpha} \quad / \quad \tilde{\alpha}_{i,j} = \begin{cases} \sum_{k=1}^{m} \kappa_{i,k} \cdot |c_{i,k}| & , \quad i = j \\ 0 & , \quad i \neq j \end{cases}$$

$$\tilde{\beta} \,/\, \tilde{\beta}_{i,j} = \begin{cases} \sum_{k=1}^{n} \tilde{\kappa}_{i,k} \cdot |\tilde{c}_{i,k}| = \sum_{k=1}^{n} \dfrac{1}{\kappa_{k,i}} \cdot |c_{k,i}| & , \quad i = j \\ 0 & , \quad i \neq j \end{cases}$$

$$\kappa_{i,k} > 0 \quad \forall \quad i,k$$

(9)

Equation 9 guarantees that the matrix τ will be, at least, nonnegative definite.

Selecting $\kappa_{i,k} = \tilde{\kappa}_{k,i} = 1$ the coefficients α and β becomes:

$$\tilde{\alpha}_{i,i} = \sum_{k=1}^{m} \left| c_{i,k} \right|, \quad \tilde{\beta}_{j,j} = \sum_{k=1}^{n} \left| c_{k,j} \right| \tag{10}$$

A more appropriate selection of the family $\{k_{i,k}\}$ can be done considering the cross-correlations coefficients:

$$\begin{aligned} \kappa_{i,k} &= \sqrt{\frac{\alpha_{i,i}}{\beta_{k,k}}} = \frac{\alpha_{i,i}}{\sqrt{\alpha_{i,i} \cdot \beta_{k,k}}} \\ \tilde{\kappa}_{k,i} &= \frac{1}{\kappa_{i,k}} = \sqrt{\frac{\beta_{k,k}}{\alpha_{i,i}}} = \frac{\beta_{k,k}}{\sqrt{\alpha_{i,i} \cdot \beta_{k,k}}} \end{aligned} \tag{11}$$

Then α and β are evaluated:

$$\begin{aligned} \tilde{\alpha}_{i,i} &= \sum_{k=1}^{m} \left| c_{i,k} \right| \cdot \kappa_{i,k} = \alpha_{i,i} \cdot \sum_{k=1}^{m} \left| c_{i,k} \right| \cdot \frac{1}{\sqrt{\alpha_{i,i} \cdot \beta_{k,k}}} \\ \tilde{\beta}_{j,j} &= \sum_{k=1}^{n} \left| \tilde{c}_{j,k} \right| \cdot \tilde{\kappa}_{j,k} = \sum_{k=1}^{n} \left| c_{k,j} \right| \cdot \frac{\beta_{j,j}}{\sqrt{\alpha_{k,k} \cdot \beta_{j,j}}} = \\ &= \beta_{j,j} \cdot \sum_{k=1}^{n} \left| c_{k,j} \right| \cdot \frac{1}{\sqrt{\alpha_{k,k} \cdot \beta_{j,j}}} \end{aligned} \tag{12}$$

Finally the diagonal coefficients are updated:

$$\begin{aligned} \alpha_{i,i} + \tilde{\alpha}_{i,i} &= \alpha_{i,i} \cdot \left(1 + \sum_{k=1}^{m} \left| \mu_{i,k} \right| \right) \\ \beta_{i,i} + \tilde{\beta}_{i,i} &= \beta_{i,i} \cdot \left(1 + \sum_{k=1}^{n} \left| \mu_{k,i} \right| \right) \end{aligned} \tag{13}$$

with $\mu_{i,k} = \dfrac{c_{i,k}}{\sqrt{a_{i,i} \cdot b_{k,k}}}$ (14)

If the states to be de-correlated have very low correlation then the correction terms $\sum_{k=1}^{m} \left| \mu_{i,k} \right|$ and $\sum_{k=1}^{n} \left| \mu_{k,i} \right|$ will be small.

4 Sub-Optimal CEKF SLAM

This section presents the integration of the de-correlation approach presented in the previous section to the CEKF algorithm. In this implementation the following assumptions are made:

1. The landmark map is created using a Relative Landmark Representation (RLR).
2. All correlations between states will be maintained except the correlations between relative landmarks that belong to different constellations. These states will be de-correlated using techniques presented in the previous section.
3. After each CEKF global update the generated cross-correlation between landmarks of different constellations will be small making the de-correlation strategy close to optimal.

Any landmark that is a base frame landmark is called 'absolute landmark' since it is represented in global coordinates. A landmark that is represented in a local frame is named 'relative landmark'.

A state is called 'absolute state' if it is a state related to the vehicle kinematics or to an absolute landmark. Any state associated to a relative landmark is called 'relative state'.

The assumptions (1) and (3) are strictly related. If the RLR map representation is used then the correlation between the states representing the relative landmarks that belong to different frames tends to be very small. Conversely, the correlation between absolute states tends to be strong especially between states of close absolute landmarks. Then it is possible to define an approach that preserves the cross-correlation of any absolute state with any other state (relative or absolute) and ignores (de-correlate) any cross-correlation between two relative states associated to two relative landmarks that belong to different constellations (defined in different local frames). The approach preserves the cross-correlation terms between relative landmarks of the same constellation.

A normal EKF full SLAM performing this de-correlation in each update will result in excessively conservative results since over-bounding will be required in each update to de-correlate. Conversely, a CEKF will only require the full update after many local updates rendering in a less conservative over-bounding strategy. According to the CEKF a global update has to be done only when the vehicle abandons a sub-region.

The landmarks states are divided in different categories according to their location with respect to the actual position of the vehicle and their frame reference. The active states group, X_a, is formed with the states of all the relative and base landmarks in the local area and the base landmarks that have relative landmarks in the local area, (X_{aR}, , X_{aB}.) . Additionally it includes the vehicle states.

The passive states group (X_b) is formed with the relative landmarks states of the constellations where the vehicle is not navigating in and all other base landmarks states.

The states of the passive relative landmarks can then be divided in two groups: group X_{bR_1} , states associated with relative landmarks that belong to the same constellation of active relative landmarks or to adjacent constellations and group X_{bR_2} , states of relative landmarks that belong to distant constellations. The cross-correlations between states X_{bR_1} with X_{bR_2} and X_{aR} with X_{bR_2} are set to zero with appropriate modification of the covariances related to X_{bR_1} and X_{aR} according to the proposed de-correlation technique. It can be seen that the only elements that needs to be maintained are contained in a band matrix of reduced size. The cross-correlation with the absolute states is also maintained.

The complete de-correllation procedure is done in each global CEKF update. In practice the most relevant information that the filter obtains is transmitted to the states corresponding to the vehicle pose, base landmarks and local relative landmarks. This implies that the memory and computation requirements will be $\sim O(N*N_a)$, assuming a constant number of landmarks N_a is used in each local region. Since N_a is << N the computation and memory requirement of the algorithm are dramatically reduced.

The implementation of the strategy proceeds as follows: The complete optimal global update is done after n CEKF internal steps on the passive states:

$$k_2 = k_1 + n$$

$$P_{ab,(k_2)} = \phi_{(k_2)} \cdot P_{ab,(k_1)}$$

$$P_{bb,(k_2)} = P_{bb,(k_1)} - P_{ba,(k_1)} \cdot \psi_{(k_2)} \cdot P_{ab,(k_1)}$$

$$X_{b,(k_2)} = X_{b,(k_1)} - P_{ba,(k_1)} \cdot \theta_{k_2}$$

In the sub-optimal CEKF the improvement of P_{bb} is ignored, then

$$P_{bb,(k_2)} = P_{bb,(k_1)} - P_{ba,(k_1)} \cdot \psi_{(k_2)} \cdot P_{ab,(k_1)} = P_{bb,(k_1)}$$

The CEKF auxiliary matrix ψ is still needed to evaluate the diagonal elements of P_{bb} to perform the de-correlation procedure. The objective is to transfer the new information to the states representing the vehicle pose, absolute landmarks and local relative landmarks, ignoring completely the covariance changes in the non-local relative landmarks states. From the viewpoint of the non-local relative landmarks, no change in their quality is obtained.

In actual SLAM applications it can be observed that the cross-correlation factors between relative landmarks of different constellations have values of order 10^{-4} or smaller. This value becomes much smaller for distant constellations. This characteristic makes the conservative de-correlation close to optimal since very small virtual noise has to be added to the relative landmarks covariances to obtain the de-correlated matrix bounds.

A less aggressive de-correlation strategy can be implemented by accepting the existence (after de-correlation) of cross-correlation between relative landmarks of different constellations provided that these constellations are geographically close. Then de-correlation is forced only between distant constellations. In the particular case when a cross-correlation factor is not small enough one of the involved states can be degraded to quality zero.

5 Results

The algorithm was implemented using the data logged with the utility vehicle shown in Figure 1. The vehicle is retrofitted with velocity and steering encoders, two lasers range sensors and GPS. The GPS is only used to obtain ground truth when operating with enough number of satellites. The vehicle operated in a large outdoor unstructured environment similar to the one shown in Figure 1.

Figure 1. Utility vehicle and outdoor environment

The final trajectory and map using the Full SLAM (CEKF) and the sub-optimal CEKF are superimposed in Figure 2 and the total estimated error in position is presented in Figure 3 . In this case an aggressive de-correlation policy was used. The filter conservatively ignores the cross-correlation between relative states that belong to different constellations, independently of weather these constellations are close or not.

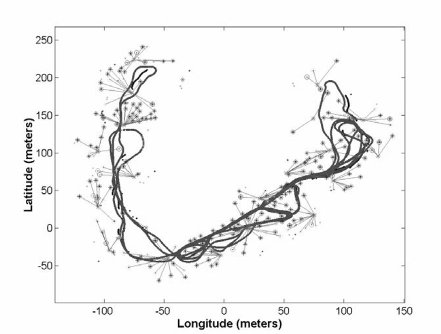

Figure 2. Final Trajectory and Map

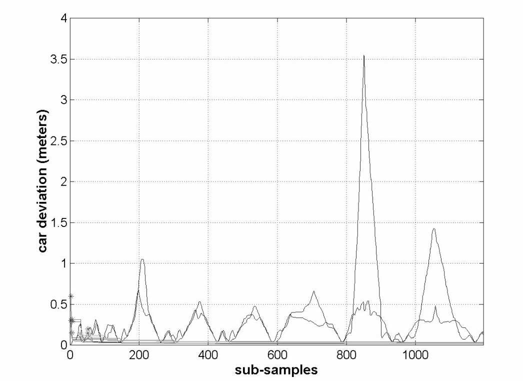

Figure 3 Estimated error for car position

Figures 4 and 5 present the position and heading difference between the optimal and sub-optimal approaches. It can be seen that the maximum discrepancy is smaller than a meter and a degree respectively.

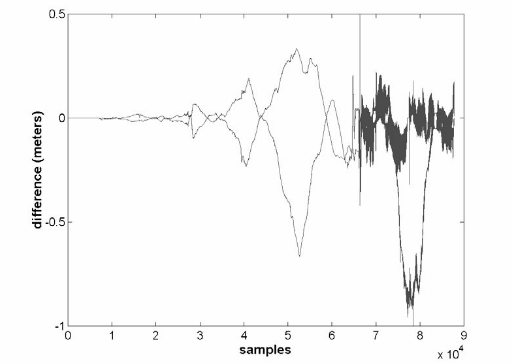

Figure 4 Difference between full SLAM and conservative approach for latitude and longitude estimation.

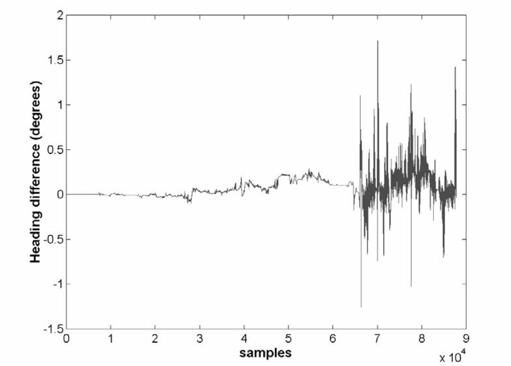

Figure 5 Difference between full SLAM and conservative approach for vehicle heading estimation.

Finally Figure 6 presents the correlation coefficient matrix. A black region represents the elements where the correlation was forced to zero.

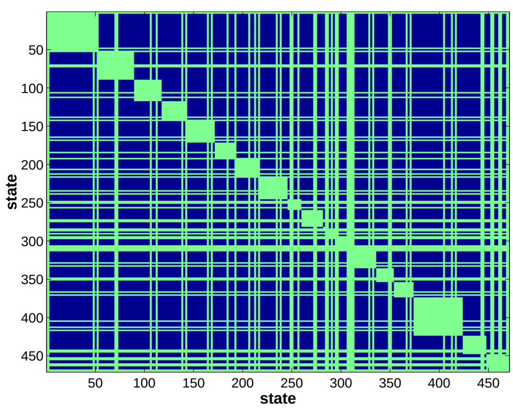

Figure 6 Correlation coefficient matrix. The black elements correspond to the coefficients that were set to zero.

6 Conclusions

This paper presents an approach to reduce the computational requirement of SLAM algorithms from $O\sim(N^2)$ to approximately $O\sim(N*N_a)$, being N_a the states representing the landmarks in the local area. With this implementation the memory requirement are also reduced to order $N*N_a$. Since N_a is $<<$ N the computation and memory requirement of the algorithm are dramatically reduced.

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